MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

ETM4096 - DIGITAL SIGNAL PROCESSING

(All sections / Groups)

6 JULY 2023
9.00 a.m - 11.00 a.m.
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 10 pages (including the cover page) with 4 questions and 6-page list of formulas given in the appendix.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

QUESTION 1

(a) A discrete-time signal $x[n]$ is shown in Figure Q1:

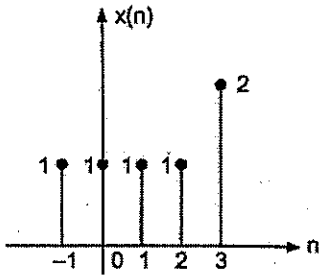


Figure Q1

Sketch the following signals:

i. $x[n-2]$ [2 marks]

ii. $x[n+1]$ [2 marks]

iii. $x[3-n]$ [4 marks]

iv. $x[n]u[n-1]$ [4 marks]

v. $x[n-1]\delta[n-1]$ [5 marks]

(b) The following is the impulse response of a discrete-time system:

$$h[n]=\left(\frac{1}{3}\right)^{n}u[-n]$$

Determine analytically whether the system is causal. [3 marks]

(c) Given a linear constant coefficient difference equation:

$$y[n]-y[n-1]+0.25y[n-2]=x[n]-0.25x[n-1]$$

i. Determine the frequency response of an LTI system whose input and output satisfy the difference equation.

[3 marks]

ii. Find the response of the system $Y\left(e^{j\omega}\right)$ to input:

$$x[n]=(0.25)^{n}u[n]$$

[2 marks]

Continued...

QUESTION 2

(a) Consider an LTI system that is characterized by the following difference equation:

$$y[n]-\frac{5}{2}y[n-1]+y[n-2]=x[n]-x[n-1]$$

i. Determine the system function $H(z)$.
ii. Realize the system function $H(z)$ in direct form II implementation.

[3 + 5 marks]

If the system is causal, answer the following questions:

iii. State the region of convergence (ROC).
iv. Draw the pole-zero plot.
v. What is the system impulse response, $h[n]$? (Hint: use partial fractions)
vi. State, with justification, if the system is realizable.

[1 + 3 + 7 + 2 marks]

(b) Suppose the realization of a digital filter in direct-form II structure is depicted in Figure Q2 below:

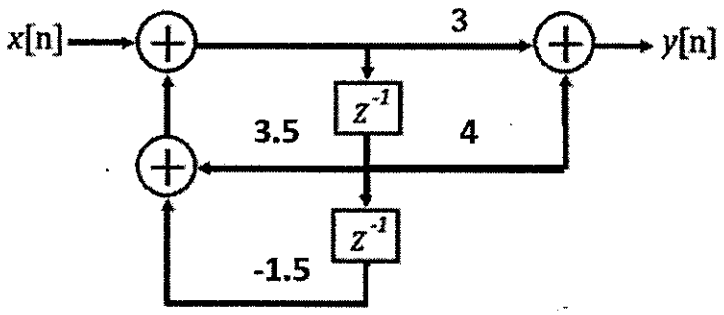


Figure Q2

Determine the transfer function, $H(z)$ of the filter.

[4 marks]

Continued...

QUESTION 3

(a) Design a low-pass FIR filter based on the follow specification:

$$0.99 \le |H(e^{j\omega})| \le 1.01 \qquad 0 \le |\omega| \le 0.19\pi$$
$$|H(e^{j\omega})| \le 0.01 \qquad 0.21\pi \le |\omega| \le \pi$$

i. Select an appropriate fixed window function that meets the given specification.
ii. Compute the order of the low-pass FIR filter.
iii. Determine the impulse response, $h[n]$, of the low-pass FIR filter.

[4 + 4 + 5 marks]

(b) Based on the following filter specification:

1. pass-band edge: $F_p = 2KHz$
2. stop-band edge: $F_s = 5KHz$
3. pass-band ripple: $\delta_p = 0.01$
4. stop-band ripple: $\delta_s = 0.01$
5. Sampling frequency: $F_T = 20\,KHz$

Assume $T = 1s$, calculate the minimum order of the Butterworth filter using:

i. Bilinear transformation
ii. Impulse-invariant method

[7 + 5 marks]

QUESTION 4

(a) Given $x[n] = \{4, 0, 4, 0\}$, determine 4-point DFT using DIT-FFT algorithm. Validate your answers by using matrix approach.

[13 marks]

(b) Verify the circular convolution y_c for the following sequence of $x[n]$ and $h[n]$:

$$x[n] = \{0,1,2,3\} \;;\; h[n] = \{2,3,4,2\}$$
$$\therefore y_c = \{19,18,13,16\}$$

[12 marks]

End of Page

APPENDIX

SUMMARY OF USEFUL TABLES

TABLE A1. DISCRETE-TIME FOURIER TRANSFORM PAIRS

Sequence	Discrete-Time Fourier Transform
$\delta[n]$	1
$\delta[n-n_o]$	$e^{-j\omega n_o}$
$1 \ (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega}+r^2e^{-j\omega})}$
$\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \omega_c < \lvert\omega\rvert < \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & otherwise \end{cases}$	$\frac{\sin[\omega(M+1)/2)]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{n=-\infty}^{\omega} 2\pi\delta(\omega-\omega_0+2\pi k)$
$\cos(\omega_0 n+\phi)$	$\sum_{n=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega-\omega_0+2\pi k)+\pi e^{-j\phi}\delta(\omega+\omega_0+2\pi k)]$

TABLE A2. SOME COMMON z-TRANSFORM PAIRS

The z-transform

Properties of the z-transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n]+by[n]$	$aX(z)+bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n]*y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z-transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n-n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

TABLE A3. SOME COMMON WINDOWS FOR FIR FILTER DESIGN

Ideal lowpass filter, n=0, 1, 2 …., N (Filter order given by N)

$$h_d[n]=\frac{\sin\left(\omega_c\left(n-N/2\right)\right)}{\left(n-N/2\right)\pi}$$

Rectangular	$w[n]=\begin{cases}1 & 0\le n\le N\\ 0 & else\end{cases}$
Hanning	$w[n]=\begin{cases}0.5-0.5\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Hamming	$w[n]=\begin{cases}0.54-0.46\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Blackman	$w[n]=\begin{cases}0.42-0.5\cos\left(\frac{2n\pi}{N}\right)+0.08\cos\left(\frac{4n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Kaiser	$w[n]=\frac{I_0\left(\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)}$ $0\le n\le N$ $\alpha=N/2$ where $\beta=\begin{cases}0.1102(A-8.7), & A>50\\ 0.5842(A-21)^{0.4}+0.07886(A-21), & 21\le A\le 50\\ 0, & A<21\end{cases}$ $N=\begin{cases}(A-7.95)/(2.285\Delta\omega), & A\ge 21\\ 5.655/\Delta\omega, & A<21\end{cases}$

TABLE A4. COMPARISON OF COMMONLY USED WINDOWS

Type of Window (Filter length = N+1)	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Stopband Attenuation, $20\log_{10}\delta$ (dB)	Transition Width (Δf)
Rectangular	-13	$4\pi/(N+1)$	-21	$0.9/N$
Bartlett	-25	$8\pi/N$	-25	
Hanning	-31	$8\pi/N$	-44	$3.1/N$
Hamming	-41	$8\pi/N$	-53	$3.3/N$
Blackman	-57	$12\pi/N$	-74	$5.5/N$

TABLE A5. IIR FILTER DESIGN

TABLE A5.1. THE COEFFICIENTS IN THE SYSTEM FUNCTION OF A NORMALIZED BUTTERWORTH LOW PASS FILTER (Ω_c=1) FOR ORDERS $1 \le N \le 8$

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.0000							
2	1.4142	1.0000						
3	2.0000	2.0000	1.0000					
4	2.6131	3.4142	2.6131	1.0000				
5	3.2361	5.2361	5.2361	3.2361	1.0000			
6	3.8637	7.4641	9.1416	7.4641	3.8637	1.0000		
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1.0000	
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1372	5.1258	1.0000

TABLE A5.2. CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad |a| < 1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

TABLE A6. PARAMETERS OF ANALOG FILTER PROTOTYPES

Discrimination factor $d = \left[\dfrac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}\right]^{1/2}$

Selectivity factor $k = \dfrac{\Omega_p}{\Omega_s}$

Butterworth	Order, $N \geq \dfrac{\log d}{\log k}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = \Omega_c e^{j(2k+1+N)\pi/(2N)}$
Chebyshev-I	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = H_a(j0) \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = -\Omega_p \sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j\Omega_p \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]$ $\epsilon = \sqrt{(1-\delta_p)^{-2}-1}$ $H_a(j0) = \begin{cases} 1, & \text{odd } N \\ (1+\epsilon^2)^{-0.5}, & \text{even } N \end{cases}$
Chebyshev-II	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{s_k}{s'_k}\left(\dfrac{s - s'_k}{s - s_k}\right)$ where $s_k = \Omega_s\{-\sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]\}^{-1}$ $s'_k = j\Omega_s / \cos((2k+1)\pi/(2N))$ $\epsilon = 1/\sqrt{\delta_s^{-2}-1}$

Elliptic	$N \geq \dfrac{\log(16/d^2)}{\log(1/q)}$, $q = q_o + 2q_o^5 + 15q_o^9 + 150q_o^{13}$, $q_o = \dfrac{1}{2}\left[\dfrac{1-(1-k^2)^{1/4}}{1+(1-k^2)^{1/4}}\right]$

TABLE A7. DISCRETE-TIME FOURIER ANALYSIS

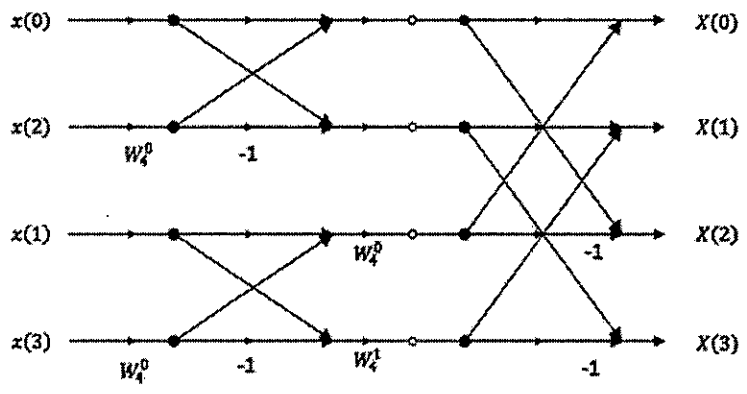
The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

DFT in Matrix Format

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

The decimation-in-time FFT



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